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Absorption cross section of smeared $D3$-brane on a circle

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Abstract: The evaluation of the absorption cross section of a massless scalar field propagating on a non-extremal black D3-brane smeared on a circle is considered. The solution to the scalar field equation of motion at high temperature is obtained in terms of the parameters $\lambda = 3\omega/4\pi T$ and k, which denote the frequency and the momentum along the circle respectively while T denotes the temperature of the black brane. Based on a perturbative scheme, higher order temperature corrections to the scalar absorption cross section are computed. Further, interesting analogies with the cross section of known black branes solutions are discussed.

Keywords: Black Holes, Black Holes in String Theory, p-branes.

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Contents

1. Introduction

According to Bekenstein-Hawking [1−3] seminal work the gravitational entropy of a Schwarzschild black hole in proper units, is proportional to the area spanned by its horizon namely, $S = A/4$. Although classically such black holes are stable this is not the case when quantum mechanical effects are taken into account. One can picture Hawking radiation as particles which are absorbed by the black hole while at the same time radiation is emmited to its environment. A very interesting result [4] states that the absorption cross section probability of a massless scalar field embedded on a spherically symmetric black hole in $p + 2$ dimensions is simply given by the area of the horizon. Similar studies have been performed for more general higher dimensional spacetimes [5, 6] leading to interesting results regarding the low frequency behavior of the absorption probability of neutral scalars. Additionally, it is known [7], that for non-dilatonic extremal D-branes in the s-wave approximation $\mathcal{P}_{\rm abs} = 0$ as ω approaches zero. An extensive treatment of Hawking emission rates even for higher partial waves in effective string models can be found in [8−16]. The importance of such calculations including non-extremal black D-branes extends also to evaluating transport properties of gauge field theories at strong coupling [17].

Most calculations that have been done so far including black D-branes have a horizon of spherical topology i.e. S^P. In this paper we will focus on a different type of branes the so called smeared Dp-branes. Smearing a brane along one of its transverse directions creates an electrically charged brane provided it has a direction with a translational symmetry along which the original brane is neutral. As is in ordinary D-branes one can define a near horizon limit making a connection to a dual supersymmetric Yang-Mills gauge theory living in a lower dimensional spacetime. Interesting phenomena have emerged out of studying such branes such as a Gregory-Laflamme [18, 19] type instability that has been observed and extensively investigated. For more details on the classical and thermodynamic stability of smeared branes consult [20 − 23] and references therein.

In what follows we shall give the details of the evaluation of the absorption cross section of a scalar field on a D3-smeared brane. This calculation is particularly involved since the metric is such that introduces a non zero momentum along the smeared direction. Given that, the solution to the scalar wave equation is obtained as a double perturbative expansion on ω/T and k in the high temperature regime[1]. According to our analysis we obtain the general expression of $\sigma_{\rm abs}$ for higher partial waves to the zeroth order in λ and k (as dictated by the perturbative scheme applied on the scalar field), while higher order corrections in λ are carried out in the s-wave approximation.

2. Smeared black Dp-branes

We begin by presenting some basic aspects of smeared d-branes following closely [23]. The way to obtain a smeared D-brane out of black string solution involves a series of steps. One starts with the following 11-dim uplift of a black string metric initially defined in $10-p$ dimensions

$$ds_{11}^2 = -f dt^2 + dz^2 + f^{-1}dr^2 + r^2 d\Omega_{7-p}^2 + \sum_{i=1}^{p} dx_i^2 + dy^2, f = 1 - \frac{r_0^{6-p}}{r^{6-p}} \tag{2.1}$$

where x_i stands for p flat directions, and y denotes the direction that we will apply a IIA reduction (S- duality transformation). Next, one applies a boost along the y-direction with a boost parameter $\alpha > 0$ so that charge is assigned to our solution. The final step is to perform a T-duality on every x_i direction so that one finally obtains a non-extremal Dp-brane along the z-direction

$$\begin{aligned} ds^2 &= H^{-1/2}\left(-f dt^2 + \sum_{i=1}^{p} dx_i^2\right) + H^{1/2}\left(f^{-1}dr^2 + dz^2 + r^2 d\Omega_{7-p}^2\right), \qquad (2.2) \\ H &= 1 + \frac{R^{6-p}}{r^{6-p}}\,, \quad e^{2\phi} = H^{\frac{3-p}{2}}\,, \quad A_{01\ldots p} = \coth\alpha\,(H^{-1}-1)\,, \quad R = r_0 \sinh\alpha^{\frac{2}{6-p}} \end{aligned}$$

where the metric is expressed in the string frame. One can also define the near-extremal limit of the above metric where details can be found in [23]. Close inspection of the line element eq. (2.1) reveals that it has a non-trivial spherical topology $S^1 \times S^{7-p}$. This in particular has an impact on the evaluation of the scalar wave function Φ by assigning a wave number (see k) in Φ even in the s-wave approximation. We shall expand on that by giving more details in the following section.

The thermodynamics of the spacetime at hand is shown to exhibit interesting properties such as a Gregory-Laflamme type instability. In the case examined though our main

[1]For an extensive analysis on the low temperature regime for near extremal branes see [24]

concern is to study scalar instead of gravity perturbations. Starting with the entropy (which is proportional to the area's horizon), is easy to prove that

$$S = L\frac{\Omega_{7-p}}{16\pi G}V_p r_0^{7-p}\cosh\alpha \tag{2.3}$$

where V_p is the world-volume of the brane, and with L we denote the circumference of the circle of the smeared direction. We note, that in the latter part of the paper we show, that the absorption probability of the scalar is proportional but not equal to the area of the horizon of the black brane in the low frequency regime. The temperature ascribed to the black brane is obtained through implementing $T = \frac{1}{4\pi}|dg_{tt}/dr|\sqrt{-g^{tt}g^{rr}}$ evaluated at the location of the horizon $r = r_0$

$$T = \frac{6-p}{4\pi r_0\cosh\alpha} \tag{2.4}$$

Before advancing further, it would be useful to point out that the evaluation of the absorption probability will be performed for large values of the boost factor which basically translates to working in the near extremal limit i.e. $R >> r_0$.

3. Scalar wave equation

The main task in this section is to solve the wave equation of a massless scalar field Φ which its dynamics is driven by the smeared D3-brane, in other words one has to solve

$$\Box\Phi = 0 \tag{3.1}$$

Given the isometries of the brane, it is natural to consider an ansatz that reads

$$\Phi = e^{i(\omega t + kz)}\phi(r)Y \tag{3.2}$$

where ω and k stand for the frequency and the momentum along the smeared dimension respectively and Y denotes the higher dimensional spherical harmonics on the $(7-p)$ sphere. In p-dimensions and working in the Euclidean frame the wave operator acts in the wave function as follows (s-wave approximation)

$$\Box_E\Phi = (\omega^2 H^{\frac{7-p}{2}}f^{-1} - k^2 H^{-\frac{p+1}{8}})\Phi + r^{p-7}H^{-\frac{p+1}{8}}\partial_r(r^{7-p}f\partial_r\Phi) \tag{3.3}$$

A similar although not identical equation can be written in the string frame. However, in our analysis we are restricted on smeared D3-branes so both frames correspond to the same wave equation. In addition, if higher partial waves are taken into account the wave equation for $p = 3$ reduces down to

$$\left(\omega^2 H^{\frac{1}{2}}f^{-1} - k^2 H^{-\frac{1}{2}} - l(l+3)r^{-2}H^{-\frac{1}{2}}\right)\Phi + r^{-4}H^{-\frac{1}{2}}\partial_r(r^4 f\partial_r\Phi) = 0 \tag{3.4}$$

Solving eq. (3.4) requires separating the entire region of spacetime in a far and a near horizon region in which the obtained solutions must be 'stretched' and matched in an intermediate domain. The above procedure is a very standard one, usually implemented when it is impossible to obtain a full solution in the domain where Φ is defined.

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3.1 Far region analysis

Before proceeding further in our analysis and for later convenience is better if we adopt $\rho = \omega r$ and $\rho_0 = \omega r_0$. Then eq. (3.4) reads

$$\frac{\partial^2\phi}{\partial\rho^2} + \left(4 + \frac{3\rho_0^3}{(\rho^3-\rho_0^3)}\right)\frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \left(\frac{\rho^3(\rho^3+(\omega R)^3)}{(\rho^3-\rho_0^3)^2} - \frac{l(l+3)\rho}{\rho^3-\rho_0^3} - \frac{(k^2/\omega^2)\rho^3}{\rho^3-\rho_0^3}\right)\phi = 0 \quad (3.5)$$

In the outer region, defined as $\rho >> \rho_0$ and $\rho >> (\omega R)^2$, one can determine the asymptotic from of the radial part of the wave function in terms of the rescaled radial distance ρ. Thus, the wave equation turns out to be

$$\frac{\partial^2\phi}{\partial\rho^2} + \frac{4}{\rho}\frac{\partial\phi}{\partial\rho} + \left(\frac{\omega^2-k^2}{\omega^2} - \frac{l(l+3)}{\rho^2}\right)\phi = 0 \quad (3.6)$$

eq. (3.6) is easily solved in terms of the Bessel functions of the first and the second kind

$$\phi(\rho) = \alpha\rho^{(\frac{-3}{2})}J_{l+\frac{3}{2}}\left(\frac{\sqrt{\omega^2-k^2}}{\omega}\rho\right) + \beta\rho^{(\frac{-3}{2})}Y_{l+\frac{3}{2}}\left(\frac{\sqrt{\omega^2-k^2}}{\omega}\rho\right) \quad (3.7)$$

where α, β are constants whose value is determined by the asymptotic behavior of the scalar and by proper matching in the intermediate region. Let us be more specific by recalling that the above solution described by eq. (3.7) is valid for big values of ρ. We can stretch somewhat the solution by trying to extend it in the so called matching region, $\rho_0 << (\omega R)^2 << \rho << 1$, which yields

$$\phi(\rho) \simeq \frac{\alpha}{\Gamma(l+\frac{5}{2})}\left(\frac{\sqrt{\omega^2-k^2}}{2\omega}\right)^{l+\frac{3}{2}}\rho^l - \beta\frac{\Gamma(l+\frac{3}{2})}{\pi}\left(\frac{2\omega}{\sqrt{\omega^2-k^2}}\right)^{l+\frac{3}{2}}\rho^{-l-3} \quad (3.8)$$

However, given that $\omega R << 1$ we must impose $\beta = 0$. Finally, for consistency reasons we demand the condition $\omega^2 > k^2$ to hold.

3.2 Inner region solution

The wave function in the inner region, $\rho_0 << \rho << 1$, is easy to obtain by substituting $x = \rho_0/\rho$, in which case eq. (3.5) transforms to

$$\frac{\partial^2\phi}{\partial x^2} - \frac{2+x^3}{x(1-x^3)}\frac{\partial\phi}{\partial x} + \left(\frac{\frac{\rho_0^2}{x^3}+\frac{\omega^3R^3}{\rho_0}}{x(1-x^3)^2} - \frac{l(l+3)}{x^2(1-x^3)} - \frac{k^2\rho^2/\omega^2}{x^4(1-x^3)}\right)\phi = 0 \quad (3.9)$$

In the inner region the ρ^2/x^3 term can be neglected when compared to ω^3R^3/ρ_0, resulting in

$$\frac{\partial^2\phi}{\partial x^2} - \frac{2+x^3}{x(1-x^3)}\frac{\partial\phi}{\partial x} + \left(\frac{\lambda}{x(1-x^3)^2} - \frac{l(l+3)}{x^2(1-x^3)} - \frac{k^2\rho^2/\omega^2}{x^4(1-x^3)}\right)\phi = 0 \quad (3.10)$$

where $\lambda \equiv 3\omega/4\pi T$. The definition of λ is established through a direct implementation of eq. (2.4) in the near extremal regime.

It is easy to verify that eq. (3.10) cannot be solved analytically. This fact necessitates the need to apply some kind of approximation scheme allowing us to study the behavior

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of the wave function as we vary x. Hence, the presence of the λ parameter is crucial into distinguishing whether one seeks a low-temperature ($\lambda >> 1$) [24] or a high-temperature ($\lambda << 1$) treatment of the problem. In our case we shall focus on the latter approximation scheme.

In the region close to the horizon ($x \simeq 1$), is easy to show that $\phi(x) \sim (1-x)^{\pm \frac{i\lambda}{3}}$. However, only the negative solution is kept since it corresponds to the incoming wave at the horizon. An improved ansatz is needed when moving away from $x = 1$ in which case an additional interpolating function is included

$$\phi(x) = (1-x)^{\sigma} F(x) \tag{3.11}$$

where $\sigma = -i\lambda/3$. By plain substitution of eq. (3.11) into eq. (3.10) and after a long and tedious calculation one gets

$$\begin{aligned} &\frac{\partial^2 F}{\partial x^2} + \left(\frac{2\sigma}{1-x} + \frac{(2+x^3)}{x(1-x^3)}\right)\frac{\partial F}{\partial x} \\ &\quad + \left(\frac{\sigma(\sigma-1)}{(1-x)^2} + \frac{(2+x^3)\sigma}{x(1-x)(1-x^3)} + \frac{\lambda^2}{x(1-x^3)^2} - \frac{l(l+3)}{x^2(1-x^3)} - \frac{k^2 r_0^2}{x^4(1-x^3)}\right) F = 0 \end{aligned} \tag{3.12}$$

As is easily observed, eq. (3.12) is very cumbersome possessing no solution in closed form. Moreover, one sees that there are two parameters involved. Based on that, we wish to apply a double perturbative expansion on λ, k which are taken to be small. Hence, $F(x)$ is decomposed as

$$F(x) = F^{(0)} + \lambda F^{(1)}(x) + \lambda^2 F^{(2)}(x) + k^2 G^{(2)}(x) + \dots \tag{3.13}$$

Studying the behavior of $F^{(0)}$ (zeroth order to λ and $\tilde{k}$) around $x = 0$ one gets two solutions (x^{-l}, x^{l+3}) which lead to the ansatz

$$F^{(0)}(x) = x^{(l+3)} F_1(x) + x^{-l} F_2(x) \tag{3.14}$$

The unknown functions F_1, F_2, are determined by substituting eq. (3.14) into eq. (3.12) while neglecting terms of higher order in $\lambda, \tilde{k}$. For completeness we provide the differential equation that F_1 obeys

$$y(1-y)\frac{d^2 F_1}{dy^2} + \left((2+\frac{2l}{3}) - (3+\frac{2l}{3})y\right)\frac{dF_1}{dy} - \left(1+\frac{l}{3}\right)^2 F_1 = 0 \tag{3.15}$$

where the following change of variables $y = x^3$ is made. It is easy to cheque [25], that eq. (3.15) is a hypergeometric equation of the form $F_1(x) =_2 \mathcal{F}_1(1+\frac{l}{3}, 1+\frac{l}{3}; 2+\frac{2l}{3}; x^3)$. Following a similar analysis we conclude that $F_2(x) =_2 \mathcal{F}_1(-\frac{l}{3}, -\frac{l}{3}; -\frac{2l}{3}; x^3)$.

4. Absorption cross section evaluation

We are almost ready to advance towards the main goal of the paper that is evaluating the absorption cross section probability of the scalar field. There are two distinct cases one has to examine separately. We begin with the first case where $l \neq 3n$, $n = 1, 2, 3, \dots,$.

We would like to have a regular solution at the horizon, $x = 1$, that is to say free of any logarithmic singularities. It turns out that $F^{(0)}(x) = x^{l+3}F_1(x) + x^{-l}D_l F_2(x)$, where the constant D_l is defined as

$$D_l = -\frac{\Gamma(2+\frac{2l}{3})\Gamma^2(-\frac{l}{3})}{\Gamma(-\frac{2l}{3})\Gamma^2(1+\frac{l}{3})} \tag{4.1}$$

Upon 'stretching' the solution described by eq. (3.14) to the matching region we recover

$$\phi(\rho) \simeq D_l \rho_0^{-l} \rho^l \tag{4.2}$$

Hence, eq. (3.8), (4.2) when compared to the intermediate region ($\beta = 0$) give

$$\alpha = 2^{(l+\frac{3}{2})}\Gamma\left(l+\frac{5}{2}\right) D_l \left(\frac{\sqrt{\omega^2-k^2}}{\omega}\right)^{(-l-\frac{3}{2})} \rho_0^{-l} \tag{4.3}$$

Additionally, the incoming wave from infinity reads

$$\phi_\infty(\rho) = \sqrt{\frac{2}{\pi}}\frac{\alpha}{2}\left(\frac{\omega^2}{\omega^2-k^2}\right)^{\frac{1}{4}} \rho^{-2} e^{-i(\frac{\sqrt{\omega^2-k^2}}{\omega}\rho - \frac{(l+2)\pi}{2})} \tag{4.4}$$

The wave function very close to the vicinity of the horizon reads

$$\phi(\rho) = \left(1-\frac{\rho_0}{\rho}\right)^{-\frac{i\lambda}{3}} \tag{4.5}$$

In general, the absorption probability is computed through

$$P(l) \equiv \frac{(\sqrt{-g}g^{rr})_h\left(\phi_h^* \frac{\partial \phi_h}{\partial r} - c.c\right)|_{r=r_0}}{(\sqrt{-g}g^{rr})_\infty\left(\phi_\infty^* \frac{\partial \phi_\infty}{\partial r} - c.c\right)|_{r=r_0}} \tag{4.6}$$

where all wave functions at the horizon and at infinity (as the subscripts indicate) are evaluated at the location of the horizon $r = r_0$. After a careful evaluation of the flux fraction we conclude that the absorption probability is simply given by

$$P(l) = \frac{2\pi\lambda\rho_0^3}{|\alpha|^2} \tag{4.7}$$

Finally, the absorption cross section for the l-th partial wave [16] is given as a function of the frequency ω and the quantized momentum k

$$\sigma_{\rm abs}^{(l\neq 3n)} = 2\pi^3(l+1)(l+2)(2l+3)A_{l\neq 3n}^2 r_0^{(2l+3)}\left(\omega^2-k^2\right)^{l-\frac{1}{2}}\left(\frac{\omega}{T}\right) \tag{4.8}$$

where

$$A_{l\neq 3n}^2 = \frac{|\Gamma(-\frac{2l}{3})\Gamma^2(1+\frac{l}{3})|^2}{|\Gamma(2+\frac{2l}{3})\Gamma^2(-\frac{l}{3})|^2 2^{(2l+3)}\Gamma^2(l+\frac{5}{2})} \tag{4.9}$$

The $l = 3n$, $n = 0, 1, 2, 3, \ldots$, case is treated differently since the regular at $x = 1$ function $F_2(x)$ is expressed in terms of the Legendre polynomials $P_n(x)$ as $F_2(x) \sim$

$P_n(\frac{2}{x^3}-1)$. Also, keep in mind that $F_2(x) \sim x^{-l}$ as x goes to zero. In this case the absorption cross section is

$$\sigma_{\rm abs}^{(l=3n)} = 2\pi^3(l+1)(l+2)(2l+3)\tilde{A}_{l=3n}^2 r_0^{(2l+3)}\left(\omega^2-k^2\right)^{l-\frac{1}{2}}\left(\frac{\omega}{T}\right) \tag{4.10}$$

where

$$A_{l=3n} = \frac{1}{2^{(2l+3)}|\Gamma(l+\frac{5}{2})|^2} \tag{4.11}$$

Interestingly enough, one can calculate the s-wave cross section by setting $l=0$ on eq. (4.9)

$$\sigma_{\rm abs}^{(l=0)} = 2(\frac{4}{3})^8\pi^6R^9T^5\left(\frac{\omega}{(\omega^2-k^2)^{\frac{1}{2}}}\right) \sim \frac{\tilde{A}_3}{V_3L}\left(\frac{\omega}{(\omega^2-k^2)^{\frac{1}{2}}}\right) \tag{4.12}$$

where $\tilde{A}_3$ stands for the area of the horizon.

Let us now comment on the expression of the absorption cross section in the various cases we investigated. First of all, in all cases $\sigma_{\rm abs}$ depends not only on the frequency of the scalar wave but also on the momentum along the smeared U(1) direction. A behavior like this was observed in the effective string models [8, 9] where Kaluza-Klein charged fixed scalars are associated with the inclusion of a momentum along a string. Also, similar results in black ring backgrounds have been reported recently [26]. At this point we remind the reader that the whole calculation was based on the assumption that the energy of the scalar probe field and the momentum along the smeared direction respect the following inequality $\omega^2 > k^2$. Given that, the absorption stays finite and at very low values of momenta k it acquires a fixed value which is independent of the frequency in the s-wave approximation. However, when higher partial waves are considered $\sigma \sim \omega^{2l}$ as easily seen in eq. (4.8), (4.10).

A close inspection of eq. (4.12) reveals that the cross section probability is proportional to the area of the horizon times the $\omega/(\omega^2-k^2)^{\frac{1}{2}}$ factor. We recall an important result of Policastro, Son and Starinets [17] which states that for a non-extremal D3-brane the absorption cross section in the zero frequency limit is exactly equal to the horizon's area. This statement generalizes a previous known universal result on black holes with spherically symmetric horizons [4]. As is apparent, in our case though $\sigma_{\rm abs}$ is not equal to the area of the horizon of the smeared D3-brane as $\omega \to 0$. This still holds even if $k=0$. We believe that the intricate topology of the horizon (product of spheres) of the smeared D3-brane plays a key role in the way the s-wave absorption cross section shapes up.

5. Cross section and temperature corrections

The absorption cross section calculation presented in the previous section was related to a perturbative expansion eq. (3.13) based on which the zeroth order term $F^{(0)}$ was computed. We shall attempt to compute the first order in λ correction to the cross section denoted by $F^{(1)}$ in the s-wave approximation. In principle, one can go even beyond the first order

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by considering higher corrections in λ however, the complexity of the resulted equations is daunting. In essence, the expansion we follow is

$$F(x) = 1 + \lambda F^{(1)}(x) \tag{5.1}$$

Hence, it turns out that by substituting eq. (5.1) directly into eq. (3.12) we get

$$\frac{d^2F^{(1)}}{dx^2} - \frac{2+x^3}{x(1-x^3)}\frac{dF^{(1)}}{dx} - \frac{i}{3}\left(\frac{2+x^3}{x(1-x)(1-x^3)} - \frac{1}{(1-x)^2}\right) = 0 \tag{5.2}$$

The solution to eq. (5.2) which respects regularity at the horizon reads

$$F^{(1)}(x) = -\frac{i}{3}\ln\left(\frac{1+x+x^2}{3}\right) \tag{5.3}$$

Finally, we conclude by providing the final expression of the absorption cross section which reads

$$\sigma_{\text{abs}}^{(l=0)} = 2(\frac{4}{3})^8\pi^6R^9T^5\left(\frac{\omega}{(\omega^2-k^2)^{\frac{1}{2}}}\right)\left(1-\left(\frac{\ln 3}{4\pi}\right)^2\left(\frac{\omega}{T}\right)^2+\cdots\right) \tag{5.4}$$

where the ellipsis stand for higher order temperature corrections. In the above result we have tacitly assumed that $(\omega \ln 3/4\pi T)^2 << 1$ which is in accordance with our approximation scheme which vouchsafes for the positiveness of the absorption cross section. To the next order in λ the differential equation which $F^{(2)}$ obeys reads

$$\begin{aligned}&\frac{d^2F^{(2)}}{dx^2} - \frac{2+x^3}{x(1-x^3)}\frac{dF^{(2)}}{dx} + \frac{2}{9}\left(\frac{2x+1}{1-x^3}\right) \\ &-\frac{1}{9}\frac{1}{(1-x)^2} + \frac{1}{x(1-x^3)^2} - \frac{1}{9}\ln\left(\frac{1+x+x^2}{3}\right)\left(\frac{2+x^3}{x(1-x)(1-x^3)} - \frac{1}{(1-x^2)}\right) = 0\end{aligned} \tag{5.5}$$

Even though we were not able to get the exact expression of $F^{(2)}$ as a solution of eq. (5.5), it turns out (through an asymptotic analysis) that this function is regular at the horizon and the boundary.

Although the effects of the smeared direction are nicely captured in the expression of the absorption cross section it would be interesting to study the effects of k at higher order in the perturbative expansion. For instance, it would be useful, if it is possible, to compute $G^{(2)}(x)$ so that we get an idea of the effects of the smeared direction on the problem at hand. It turns out that $G^{(2)}$ is computable and regular at the horizon

$$G^{(2)}(x) = -\frac{r_0^2}{10}\ln\left(\frac{1+x+x^2}{3}\right) - r_0^2\frac{\sqrt{3}}{15}\tan^{-1}\left(\frac{\sqrt{3}(2x+1)}{3}\right) - \frac{r_0^2}{10}\left(1-\frac{1}{x^2}-\frac{\sqrt{3}\pi}{45}\right) \tag{5.6}$$

Obviously, the function is divergent as one approaches the boundary ($\rho \to \infty$). However, if one takes the double limit $x, r_0 \to 0$, $G^{(2)}$ becomes finite. Thus, the irregularity at the boundary can be removed for small values of the momentum and for "tiny" smeared black branes. We believe that the fact that $G^{(2)}$ diverges at the boundary may signal the need to impose different boundary conditions at infinity in place of the usual Dirichlet conditions.

6. Conclusions

In this paper we fill a gap in the literature by presenting a detailed calculation of the absorption cross section of a neutral massless scalar field propagating in the vicinity of a black smeared D3-brane. We basically started from the well known technique of separating the spacetime (generated by the black brane) in several asymptotic regions. This in particular enabled us to obtain an explicit solution of the wave equation in each one of those regions.

One of the intricacies of our calculation was to develop a perturbative scheme based on the two parameters λ, k which enter the calculation. Such an implementation is shown to work well up to the first order in λ. The perturbative expansion on the momentum k was shown to be consistent with the boundary conditions on the horizon. However, the expansion to the k^2 order exhibits irregular behavior at infinity unless the black brane has vanishingly small radius. Finally, as one approaches the decompactification limit the wave function of the scalar field should be expressed only in terms of a perturbative series in λ. One might attribute those results to the non trivial topology (product of spheres) of the horizon. To begin with, the horizon has an $S^1 \times S^4$ topology however as $k \to 0$ it will resemble more like to an $R^1 \times S^4$.

Also, we pointed out that $\sigma_{\rm abs} = \sigma(\omega, k)$. Hence, σ exhibits an additional dependance on the momentum along the smeared direction, in contrast to what is known for ordinary black d-branes. However we showed that for very small values of the momentum k along the smeared direction the absorption cross section is directly proportional to the area of the horizon. Finally, higher order temperature corrections to the absorption cross section were carried out.

It would be interesting to pursue similar studies in other smeared D-brane backgrounds so that we uncover useful information about D-brane physics. It would be appealing to consider scalar perturbations of massive scalars. This would certainly complicate the calculations even further, however we might be able to see how the absorption cross section is affected by the mass term. Finally, one may also consider fermions probing the black brane and study their effects in the low temperature regime as was done for scalars in [24].

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